# Game Theory and Nash Equilibrium

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Let's start with a familiar game – Tic-Tac-Toe. To understand why we all are familiar with some of the concepts of game theory unconsciously, we would analyse the following situation.

| | | |
|---|---|---|
| X | | O |
| | X | |
| | | ? |

In this situation, the best way 'O' can respond is to tick in the question marked box. Why? This is because in the given situation, 'O' is faced with two options- either try to prevent 'X' from winning (tick the question box) or accept defeat (tick any other box). Thus, given this situation, this is the best strategy 'O' can choose.
This is basically what we would expound upon, the choosing of the best strategy. The above game can be continued, but the thought process is important. Let's first introduce some terms. Later, we will examine examples to get a feel of them.

Game theory is the study of mathematical models of strategic interactions among rational agents.[1] We say rational because often, in real life, people don't play the best optimal strategy. John von Neumann's paper 'On the Theory of Games of Strategy' in 1928, introduced it as a separate field. A non-cooperative game is where the players compete against each other and there are no alliances. We assume that,

1. Each player remembers their decisions.
2. Each player has complete information about strategies that can be taken.
3. Each player is rational.
4. Each player considers only their interests.
5. They all have the same understanding of the game.

In such games, a Nash equilibrium is when no players can gain anything by changing only their own strategy (others' strategies remain the same).

1. **Myerson; Roger B. (1991). Game Theory: Analysis of Conflict.**

We will understand what Nash equilibrium is by constructing what is called a payoff matrix in the following example.

| P1/P2 | A | B |
|---|---|---|
| A | 2,2 | 3,1 |
| B | 1,3 | 1,1 |

Suppose there are two players: P1, and P2. The strategies of each of them are A and B, with the numbers indicating the points. The vertical A and B's are P1's strategies, and the horizontal ones are P2's. Each box denotes strategy combination of P1 and P2. In any box, the first number denotes the points of P1, and the second number is that of P2. Let's analyse this.

Suppose, P2 chooses A. There are two strategies which P1 can take- A or B. P1 will choose the best strategy for oneself irrespective of what happens to P2. What is the best strategy for P1? Strategy A! Why? Suppose P1 chooses B (P2 has chosen A). Then, from the payoff matrix, we see the pointboard is 1,3 (P1 gets 1 point, P2 gets 3 points). But if P1 chooses A (keep in mind P2 has chosen A), P1 gets 2 points (the first number of 2,2) which is more advantageous. Here, each player is concerned only with their gain irrespective of what happens to other player. Thus,
If P2 chooses A, P1 chooses A. (A, A)
If P2 chooses B, P1 chooses A. (A, B) (keep in track of order!)
If P1 chooses A, P2 chooses A. (A, A) (the second number is P2's point!)
If P1 chooses B, P2 chooses A. (B, A)
The common strategy is (A, A), as seen from the above analysis. This is the Nash equilibrium.

Let's try for a little more complicated one. (Try it on your own, forming statements like the above)

| P1/P2 | A | B |
|---|---|---|
| A | 6,6 | 6,8 |
| B | 8,3 | 4,3 |

If P2 chooses A, P1 chooses B. (B, A)
If P2 chooses B, P1 chooses A. (A, B)
If P1 chooses A, P2 chooses B. (A, B)
If P1 chooses B, P2 chooses A or B. (B, A) or (B, B)
So, there can be more than one Nash equilibrium (here it's (A, B), (B, A))

Another problem:

| P1/P2 | A | B |
|---|---|---|
| A | 6,6 | 6,5 |
| B | 8,3 | 4,4 |

As you solved, there is, infact, no Nash equilibrium here.

In all these cases, what we have implicitly assumed is that we are playing pure strategies, that is we can't use a combination of strategies A and B. For large number of similar games, we can use a certain combination of A and B instead of sticking to just one. This is called mixed Nash equilibrium.

Let's consider a famous example – the Hawk-Dove game.

| P1/P2 | Hawk | Dove |
|---|---|---|
| Hawk | -1,-1 | 2,0 |
| Dove | 0,2 | 1,1 |

Doing similar analyses as above, we see (Hawk, Dove) and (Dove, Hawk) are Nash equilibrium for pure strategies.
Suppose P1 chooses Hawk strategy with probability $p$ and Dove strategy rest of the time (with probability $1-p$). P2 chooses Hawk strategy with probability $q$ and Dove strategy rest of the time (with probability $1-q$).
Then, points scored by P1 when P2 chooses,
...Hawk is $-1 \cdot p + 0 \cdot (1-p)$
...Dove is $2 \cdot p + 1 \cdot (1-p)$
We equate these two equations to get $p = 0.5$
Then, points scored by P2 when P1 chooses,
...Hawk is $-1 \cdot q + 0 \cdot (1-q)$
...Dove is $2 \cdot q + 1 \cdot (1-q)$
Similarly, $q = 0.5$
Thus, the equilibrium strategy here would be
P1 plays Hawk 50%, Dove 50%
P2 plays Hawk 50%, Dove 50%
The result isn't interesting, but for different payoffs, it may be different.

This, however, is also what is called an evolutionary stable strategy (ESS), which, when fixed in a population, natural selection will alone prevent alternative strategies from replacing it. However, it is not the same as Nash equilibrium even though, in this case, it is. It has a stricter criterion than Nash equilibrium, meaning all ESS are Nash equilibrium, but not all Nash equilibriums are ESS. We won't go into what an ESS is in this article.

Game theory, originally developed for economics, has widespread application in psychology, evolutionary biology, war, politics, and business. The last example borders on evolutionary biology, where an ESS is important in understanding animal interactions, first applied by John Maynard Smith. You can check his book 'Evolution and the Theory of Games' for more on ESS.